Published by Institute of Physics Publishing for SISSA/ISAS
Received: *May 11, 2004*
Accepted: *September 6, 2004*

Charged black holes from near extremal black holes

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Abstract: We recover the properties of a wide class of far from extremal charged black branes from properties of near extremal black branes, generalizing the results of Danielsson, Guijosa and Kruczenski.

Keywords: D-branes, Black Holes in String Theory.

 http://jhep.sissa.it/archive/papers/jhep092004009/jhep092004009.pdf

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1. Introduction

The underlying physics of black holes have been the target of many investigations. It has been realized that certain near extremal black holes can be modeled [1] by a large number of D-branes with excitations on them. This spurred the description of string theory in certain backgrounds by a large N gauge theory in the strong 't Hooft coupling regime [2, 3]. It seemed that some new physics has to come in, in order to describe the far from extremal and Schwarzschild black holes.

Recently generalizing the construction in [4] a possibility of understanding Schwarzschild black holes in various dimensions [5, 6, 7]and rotating neutral black holes [8, 5] in various dimensions from the physics of near extremal black hole have emerged. Since the physics of near extremal black holes is governed by large N gauge theories, this means the the physics of far from extremal black holes can be similarly understood. In particular the understanding of the relationship between area and entropy [9, 10], and the origin of the horizon size [11] for the near extremal case , goes over to the Schwarzschild black brane case as well. In this note we generalize this description to the far from extremal charged black holes case (or black branes). The example of the single charged black brane in seven dimensions was constructed in [4],[1] and we generalize it here to any dimension and to the multi charged case.

2. Single charged black brane

We start with the simplest case where the black branes carry only one kind of charge from the ten dimensional perspective. i.e they are made from configurations with different numbers of D-branes and anti D-branes, of the same dimensionality.

[1] See also [12].

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The mass, entropy and charge of the non extremal charged p-brane in D-dimensions we wish to understand are given by a two parameter family [13] ($D = p + d + 3$)

$$\begin{aligned} M_{bh} &= \frac{\omega_{d+1}}{2\kappa^2} L^p \mu^d (d + 1 + d \sinh^2 \gamma) \\ S_{bh} &= \frac{2\pi}{\kappa^2} \omega_{d+1} L^p \mu^{d+1} \cosh \gamma \\ Q_{bh} &= \frac{1}{2\sqrt{2}\kappa} \omega_{d+1} L^p \mu^d \sinh 2\gamma \,. \end{aligned} \tag{2.1}$$

The near extremal entropy of these black holes is given by [14]

$$\begin{aligned} S &= a \left(\frac{E}{2}\right)^{\lambda} \sqrt{M_{p,0}} \\ \lambda &= \frac{d+1}{d} - \frac{1}{2}, \qquad D = p + d + 3 \\ a &= 2^{\frac{1}{d}+2} \pi \omega_{d+1}^{-\frac{1}{d}} d^{-\frac{d+1}{d}} \lambda^{-\lambda} L^{-\frac{p}{d}} \kappa^{\frac{2}{d}} \\ M_{p,0} &= \frac{q_p}{\sqrt{2}\kappa} L^p \,, \end{aligned} \tag{2.2}$$

where q_p is the charge per unit volume and E is the energy above extremality.

We will assume that the thermodynamics of the brane and anti brane system is unaffected by the presence of the other, and is controlled by the thermodynamics of the near extremal branes, even when E is large. One can then try and maximise the entropy of the brane anti-brane system to read off the parameters of the system. If we do this, one condition is that the temperature of the excitations on the brane is equal to the temperature of the excitations on the anti brane leading to an incorrect result. Now usually this is a natural condition, when two gases of excitation are in thermodaynamical contact. This however is not the case here. The stability of the system means that the strings connecting the brane and anti-brane are actually massive and since the temperature on the branes and anti branes will later seen to be small this means that the two gases of excitations on the brane and anti brane are effectively (after stabilization) decoupled. What seems to work is requiring that the energy of the gas of excitation on the brane and anti-brane be equal (in the Schwarzschild case this is the same as equall temperatures). We believe this condition arises from the condition of stability of the system, but can not currently derive this. Note however that this indicates that the energy needed for stabilization is a function of the number of tachyonic modes, which is equall on both configurations.

We will thus just proceed with this assumption. Let us label the mass of the branes (anti-branes) by $M_{p,0}(M_{\bar{p},0})$, and the energy of excitation on both branes by E. Then we write

$$\begin{aligned} M_f &= M_{p,0} + M_{\bar{p},0} + E \\ S_f &= a \left(\frac{E}{2}\right)^{\lambda} \left(\sqrt{M_{p,0}} + \sqrt{M_{\bar{p},0}}\right) \\ q_f &= \frac{\sqrt{2}\kappa}{L^p} (M_{p,0} - M_{\bar{p},0}) \,. \end{aligned} \tag{2.3}$$

We now maximise the entropy with respect to $M_{p,0}$ (basically the number of branes) while keeping the mass and charge fixed. This gives the relationship

$$E = 4\lambda\sqrt{M_{p,0}M_{\bar{p},0}}\,. \tag{2.4}$$

We can plug this back to the equation for the mass and charge and then solve for $M_{p,0}$. If the mass and charge are given by equation (2.1) then the solution of maximising the entropy gives

$$\begin{aligned}
M_{p,0} &= \frac{\omega_{d+1}}{2\kappa^2}L^p\mu^d\frac{d}{4}e^{2\gamma} \\
M_{\bar{p},0} &= \frac{\omega_{d+1}}{2\kappa^2}L^p\mu^d\frac{d}{4}e^{-2\gamma} \\
E &= \lambda\frac{\omega_{d+1}}{2\kappa^2}L^p\mu^d d\,.
\end{aligned} \tag{2.5}$$

We can now plug these expression into the expressions of the entropy in equation (2.3) and we find

$$S_f = 2^{-\lambda}S_{bh}\,. \tag{2.6}$$

So up to a proportionality constant for the entropy we have reproduced the dependence of the entropy on the charge and mass of the far from extremal charged black brane.

3. Multi brane black branes

In this section we will generalize the above construction to the situation where there are more than one type of branes involved, but in some lower D-dimension it can be represented as a single charged black brane. We want to reproduce the properties of the black branes discussed in [13] (for a general N),

$$\begin{aligned}
M_{bh} &= \frac{\omega_{d+1}}{2\kappa^2}L^p\mu^d(d+1+dN\sinh^2\gamma) \\
S_{bh} &= \frac{2\pi}{\kappa^2}\omega_{d+1}L^p\mu^{d+1}\cosh^N\gamma \\
Q_{bh} &= \frac{\sqrt{N}}{2\sqrt{2}\kappa}\omega_{d+1}L^p\mu^d\sinh 2\gamma\,.
\end{aligned} \tag{3.1}$$

The near extremal entropy is given by [14]

$$\begin{aligned}
S(E) &= b' q_p^{\frac{N}{2}}E^\lambda \\
b' &= 2^{\frac{1}{d}-\frac{N}{4}+2}\pi\omega_{d+1}^{-\frac{1}{d}}d^{-\frac{d+1}{d}}\lambda^{-\lambda}N^{-\frac{N}{4}}L^{p(1-\lambda)}\kappa^{\frac{2}{d}-\frac{N}{2}} \\
M_{p,0} &= \frac{q_p\sqrt{N}}{\sqrt{2}\kappa}L^p\,, \qquad \lambda = \frac{d+1}{d}-\frac{N}{2}
\end{aligned} \tag{3.2}$$

Where N represents the number of diffrent D-branes. Now there is a question what is the near extremal entropy of an object made out of the black brane and anti black brane. When a single brane was involved the brane anti brane is given by the equation in (2.3), but now things are different. To see this let us imagine we are considering the configuration of the $D1-D5$-branes. The low energy states are given by the $D1-D5$ strings while the $D1-D1$ and $D5-D5$ are presumed massive. If we now take a system with a $D1-D5$ and an $\bar{D}1-\bar{D}5$, there are four sectors of low energy excitations, which include also the $\bar{D}1-D5$ and the $D1-\bar{D}5$ strings. If we consider the situation in which in six dimensions the charge of the $D1$ is equal the charge of the $D5$ then the near extremal mass and entropy of the system can be written as

$$\begin{aligned}
M_f &= M_{p,0} + M_{\bar{p},0} + E \\
S_f &= a\left(\frac{E}{2^N}\right)^{\lambda} (\sqrt{M_{p,0}} + \sqrt{M_{\bar{p},0}})^N \\
a &= b'\left(\frac{\sqrt{2}\kappa}{L^p}\right)^{N/2} \\
q_f &= \frac{\sqrt{2}\kappa}{\sqrt{N}L^p}(M_{p,0} - M_{\bar{p},0})\,,
\end{aligned} \tag{3.3}$$

where the energy was equally distributed among all 2^N kinds of low energy excitations. We now can maximise the entropy with respect to $M_{p,0}$ keeping the mass and charge fixed, and we find the condition,

$$E = \frac{4\lambda}{N}\sqrt{M_{p,0}M_{\bar{p},0}}\,. \tag{3.4}$$

We can now again plug into the mass and charge to find

$$\begin{aligned}
M_{p,0} &= \frac{\omega_{d+1}}{2\kappa^2}L^p\mu^d\frac{Nd}{4}e^{2\gamma} \\
M_{\bar{p},0} &= \frac{\omega_{d+1}}{2\kappa^2}L^p\mu^d\frac{Nd}{4}e^{-2\gamma} \\
E &= \lambda\frac{\omega_{d+1}}{2\kappa^2}L^p\mu^d d\,.
\end{aligned} \tag{3.5}$$

This can now be plugged into the expressions for entropy equation (3.3) and we find

$$S_f = 2^{-N\lambda}S_{bh}\,. \tag{3.6}$$

Again we reproduce the entropy mass (and charge) relationship up to a numerical constant.

4. Multi charged black holes

We now come to a more general situation where the different D-branes carry diffrent charges. The black holes are described in [15] and their mass and entropy (upon some re labeling) are given by

$$\begin{aligned}
M_{bh} &= \frac{b}{2}\mu^{D-3}\left(\sum_{i=1}^{N}\cosh\gamma_i + 2\lambda\right) \\
b &= \frac{\omega_{D-2}}{2\kappa^2}(D-3)V_p\,, \qquad \lambda = \frac{D-2}{D-3} - \frac{N}{2} \\
S_{bh} &= c\mu^{D-2}\Pi_{i=1}^{N}\cosh\gamma_i \\
c &= \frac{4\pi b}{D-3}\,.
\end{aligned} \tag{4.1}$$

The near extremal entropy is given by [16]

$$\begin{aligned} S &= \tilde{a} E^{\lambda} \Pi_{i=1}^{N} \sqrt{M_{p_i,0}} \\ \tilde{a} &= c(b\lambda)^{-\lambda} b^{-N/2} \\ M_{p,0} &= b q_p \,. \end{aligned} \tag{4.2}$$

Following the same logic as before we can try and model the far from extremal black hole using the near extremal relationship. After taking into account all configurations of brane that contribute to the entropy, we can write,

$$\begin{aligned} M_f &= \sum_i^N (M_{p_i,0} + M_{\bar{p}_i,0}) + E \\ S_f &= \tilde{a} (\frac{E}{2^N})^{\lambda} \Pi_{i=1}^{N} \left(\sqrt{M_{p_i,0}} + \sqrt{M_{\bar{p}_i,0}} \right) . \end{aligned} \tag{4.3}$$

Maximizing the entropy for fixed charge and mass gives the relationship

$$E = 4\lambda \sqrt{M_{p_i,0} M_{\bar{p}_i,0}} \,, \tag{4.4}$$

for all i.

Solving the equations gives

$$\begin{aligned} M_{p_i,0} &= \frac{b}{4} \mu^{D-3} e^{2\gamma_i} \\ M_{\bar{p},0} &= \frac{b}{4} \mu^{D-3} e^{-2\gamma_i} \\ E &= b\lambda \mu^{D-3} \,. \end{aligned} \tag{4.5}$$

Plugging it back to the expression for the entropy equation (4.3) gives the entropy of the far from extremal charged black hole, equation (4.1), up to a numerical factor of $2^{-\lambda N}$. Note that in all cases, as noticed in [4], the entropy would be exactly the correct one if the energy on each brane configuration would have been the total energy.

5. Horizon and temperature

In this section we would like to briefly discuss some physical properties of the brane system. If we look at equations (2.5), (3.5) and (4.5), we see that the dependence of E on the parameter of the solutions (i.e. μ and γ) $E \sim \mu^d$ (or μ^{D-3}) are exactly the same as the dependence of the energy of the near extremal black hole on these parameters [14, 16]. Now since in the coordinate system used in [14, 16] μ is also the radius of the horizon, this means that the emergence of a large horizon size has its origin in the same physical phenomena as the emergence of the near extremal horizon . Thus again as in the Schwarzschild case the charged black brane horizon can be understood as the regulated transverse size of the gauge theory state [17, 11], and can be seen by a brane probe as the locus of the tachyonic instability region [18, 19].

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While the size of the state given by the branes or anti branes is the same (it is only a function of the energy of the excitations on them) the temperature is different. The relationship between the temperature of the black brane and the field theory temperature on each brane configuration just follows from the regular thermodynamical relationship

$$\frac{1}{T_{bh}} = \frac{\partial S}{\partial M} = \frac{\partial S}{\partial M_{p_i,0}} \frac{\partial M_{p_i,0}}{\partial M} + \frac{\partial S}{\partial E}\frac{\partial E}{\partial M} = 2^{-N} \sum_{i=1}^{N} \frac{1}{T_i}\,, \tag{5.1}$$

where in the last equality we have used that since the entropy was maximal,

$$2\frac{\partial S}{\partial E} = \frac{\partial S}{\partial M_{p_i,0}}\,. \tag{5.2}$$

The appearance of more that one temperature should have some signal in the supergravity properties of these black holes, at least through the properties of the quasinormal mode. The temperatures on each brane configuration should be low for the approximation we have been using to be justified. This means that we should be quite far from extremality (γ_i's should not be too big). As one approaches extremality at least one of the temperatures becomes high, and in particular can become high enough to excite the massive modes between brane and anti-brane, this can lead to the disappearance of the branes or anti-branes through annihilation.

Acknowledgments

I thank O. Bergman for useful discussions. This work is supported in part by the US-Israel Binational Science Foundation grant no. 2000359.

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